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INITIAL MIXING OF THERMAL DISCHARGES INTO A UNIFORM CURRENT

John Eric Edinger, et al

National Center for Research and Training
in the Hydrologic and Hydraulic Aspects of
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by

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and

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National Center For Research and Training in the
Hydrologic and Hydraulic Aspects of Water Pollution Control

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TABLE OF CONTENTS

	Page
INTRODUCTION	1
TWO DIMENSIONAL CASE	5
Conservative Case	5
Analytic Development	5
Graphical Summary for Streams	8
Area Within a Concentration Contour	9
Non-Conservative Case	13
Analytic Development	15
Application to Streams	16
Area Within a Concentration Contour	17
THE THREE DIMENSIONAL CASE	21
Surface Area Within a Concentration Contour	22
Cross-Sectional Area Within Isoconcentration Contour	25
SUMMARY OF PROPERTIES FOR INITIAL TEMPERATURE DISTRIBUTIONS FROM SIMPLE HEAT BALANCES	29
Properties of Two Dimensional Distributions	29
Properties of Three Dimensional Distributions	30
CASE COMPARISONS WITH FIELD DATA	32
Discharge Into Open Water	32
River Discharge Analysis	35
ESTIMATING DIFFUSIVITIES	43
REFERENCES	45

LIST OF FIGURES

Figure		Page
1	CONCENTRATION RATIO TO FULLY MIXED CONCENTRATION AS A FUNCTION OF THE NON-DIMENSIONAL DOWNSTREAM COORDINATE FOR SELECTED VALUES OF THE NON-DIMENSIONAL WIDTH	10
2	NON-DIMENSIONAL SURFACE AREA RATIO AS A FUNCTION OF CONCENTRATION RATIO FOR THE TWO DIMENSIONAL CASE	14
3	RELATIONSHIP OF TEMPERATURE RISE RATIO TO NON-DIMENSIONAL SURFACE AREA RATIO FOR SELECTED VALUES OF β	20
4	NON-DIMENSIONAL SURFACE AREA RATIO AS A FUNCTION OF CONCENTRATION RATIO FOR THE TWO DIMENSIONAL AND THREE DIMENSIONAL CASES	24
5	TEMPERATURE RISE RATIO VERSUS SURFACE AREA WITHIN THE TEMPERATURE RISE CONTOUR FOR THE DATA OF CHENEY AND RICHARDS	34
6	TEMPERATURE RISE RATIO VERSUS SURFACE AREA WITHIN THE TEMPERATURE RISE CONTOUR FOR DATA FROM WIDOWS CREEK STEAM PLANT	38
7	RATIO OF TEMPERATURE RISE TO FRACTION OF CROSS-SECTIONAL AREA WITHIN THE TEMPERATURE RISE CONTOUR AT EACH DOWNSTREAM CROSS-SECTION	40
8	RELATIONSHIP OF $\frac{1}{g(x)}$ AND DISTANCE DOWNSTREAM, L, BELOW WIDOWS CREEK STEAM PLANT	42

LIST OF TABLES

Table		Page
1	SURFACE TEMPERATURE INFLUENCE MEASURED AT MORRO BAY POWER PLANT	33
2	SURFACE AREA TEMPERATURE DATA FROM WIDOWS CREEK STEAM PLANT	36
3	CROSS-SECTIONAL AREA TEMPERATURE DATA FROM WIDOWS CREEK STEAM PLANT	37
4	LATERAL AND VERTICAL DISPERSION COEFFICIENTS DETERMINED FROM WIDOWS CREEK TEMPERATURE DATA	41

INTRODUCTION

There is an immediate need for simple analytic descriptions of the temperature distributions which result from the initial dispersion of a heated water discharge into a waterbody. A few questions related to these discharges are : (1) how does the distribution depend on plant size?; (2) how are they related to characteristics of the receiving waterbody?; (3) what are the quantitative differences between two-dimensional and three-dimensional temperature distributions?; (4) what is the influence of surface cooling on initial distributions?

There are a number of possible approaches to examining these questions ranging from empirical development based on field data to numeric simulation of the combined conservation and hydrodynamic equations. The approach used in this study is to examine analytic solutions to the basic heat conservation equation which might apply to heated discharges. This development suggests possible empirical representations of field data as well as provides a background for further numeric and analytic studies.

There are a number of reasons for investigating simplified analytic solutions (as opposed to generalized numeric solutions) at this stage of research:

1. There is a need for simple generalized tools for rapid but approximate initial computations and preliminary examination of field data.

2. Study of applicability, or inappropriateness, of simple analytic solutions to initial mixing problems has not yet been examined in a systematic manner.

3. It is necessary to have simplified analytic solutions to test convergence of more generalized numeric methods.

4. To develop insight into the relationships between the many variables entering the heat conservation equation and their influence on descriptions of initial temperature distributions.

5. To provide a systematic method for physical and analytic classification of discharges.

6. To find possible empirical descriptions of initial discharge conditions for comparison to field data.

It has been attempted to maintain a balance between the oversimplification of the analytic solutions and properties of the temperature field being described. Two useful descriptions have evolved: (a) the fraction of a cross-sectional area contained within a given temperature rise contour as a function of distance from the discharge and (b) the surface area contained within a given temperature rise contour.

Both of these " contour area" descriptions result in equations that have parameters of the velocity field and dispersive field grouped as multiplier or scaling terms that can be evaluated from field data. Working with areas within contours eliminates the complications of identifying plume axes and coordinate positions and provides an empirical simplification of the complex problem of non-symmetric distributions and cross-current discharges.

The first description, as the fraction of cross-sectional area, provides a simple and rapid method of reducing field data by contouring which helps average the data and eliminates a point-by-point trial and error comparison. It also provides a simple prediction tool which can be applied to a case with rapid computation.

The second description, as surface area within contours, provides an additional, but not independent, method of fitting field data and

provides a check on accuracy of data reduction procedures. It is also directly related to analysis of aerial survey and surface infrared data.

The influence of dimensionality is examined by considering two-dimensional (lateral and longitudinal) distributions and three dimensional distributions as distinct catagories. The two-dimensional case can be examined for both conservative and non-conservative properties with surface area within a concentration (temperature rise) contour as the basic description. The three-dimensional distribution does not have a simple analytic solution for the non-conservative case but is described by both surface area within a concentration contour and fraction of cross-section within a concentration contour as a function of distance from the discharge.

The basis for classification and simplification of initial temperature distributions is the three-dimensional conservation equation commonly written for a unit mass within a waterbody as:

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial x}\left(D_x\frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial_z}\left(D_z\frac{\partial s}{\partial z}\right) + \frac{h}{\rho c_p} \quad (1)$$

where s is the concentration per unit mass (or in the case of heat, the temperature rise); u, v, and w, the longitudinal, lateral and vertical velocity components at a point, x, y, z, at time t; D_x, D_y, and D_z, the longitudinal, lateral, and vertical dispersion coefficients at a point and for a given time; and h, an internal sink or source term giving the rate at which heat enters or leaves the unit mass such as from attenuation of short-wav. radiation below the water surface.

Any contaminant distribution model based on the basic conservation equation requires some specification of the velocity field for incorporation in the advection terms and specification of the terms describing dispersion. Conceptually, velocity distributions within a waterbody could be determined from the hydro-dynamic equations and when developed for a generalized set of conditions would make the additional step of solving the contaminant distribution equation mathematically simple. The major assumptions in the development of contaminant distribution models relate to simplifying the description of the velocity field and dispersion parameters within a receiving waterbody.

THE TWO-DIMENSIONAL CASE

The simplest type of initial distribution results from the lateral dispersion of heat into a uniform longitudinal velocity field with no vertical gradient of temperature. This case will be considered first to illustrate the simplifying assumptions involved and the properties of analytic descriptions which result. It is used as a bases for considering the influence of surface cooling on the initial distribution of heat by comparing the conservative distribution with the non-conservative distribution.

Conservative Distribution

The conservative distribution in the two-dimensional case results from considering the heat balance at a point between the rate of change in longitudinal advection and rate of lateral dispersion as:

$$u \frac{\partial s}{\partial x} = \frac{\partial}{\partial y}(D_y \frac{\partial s}{\partial y}) \qquad (2)$$

Analytic development - For a uniform velocity field and constant dispersion coefficient this balance can be expressed analytically by the normal distribution function as:

$$s(x, y) = \frac{C}{2\sqrt{\xi}} e^{\frac{-y^2}{4\xi}} \qquad (3)$$

where C is a constant of integration to be determined from boundary conditions and ξ is a convenient longitudinal coordinate given by:

$$\xi = \frac{x D_y}{u} \tag{4}$$

The constant of integration, C, is evaluated by considering the source discharge to be located at a point $\xi = \xi_s$, $y = 0$ and having a concentration at this point of $s = s_p$. This gives:

$$\frac{s}{s_p} = \left(\frac{\xi_s}{\xi}\right)^{1/2} e^{-\frac{y^2}{4\xi}} \tag{5}$$

The source coordinate position, ξ_s, is uniquely determined by equating the source strength, S, with the rate of advection through a downstream cross-section as:

$$S = ud \int_b sdy \tag{6}$$

where $\int_b$ represents the integral between lateral boundaries and the depth, d, of waterbody. The relations to follow will be developed for a source located on a shoreline parallel to the direction of longitudinal velocity and ignoring reflection at a far bank. Inserting Equation 5 into Equation 6 gives:

$$S = uds_p\sqrt{\xi_s} \int_0^\infty \frac{e^{-\frac{y^2}{4\xi}}}{\sqrt{\xi}} dy \tag{7}$$

The integral is a definite integral independent of ξ and has a value of $\sqrt{\pi}$. This gives the relationship between source strength, S, source concentration, s_p, and source position ξ_s of:

$$\xi_s = \frac{1}{\pi} \left(\frac{S}{uds_p}\right)^2 \tag{8}$$

For power plant discharges, the source strength and concentration are related as:

$$\frac{H_p}{\rho c_p} = S = \Theta_p Q_p \quad ; \qquad s_p = \Theta_p \tag{9}$$

where H_p is the rate of heat rejection ($BtuDay^{-1}$); ρ and c_p, the specific weight and specific heat of water; Θ_p, the condenser temperature rise; and Q_p, the condenser pumping rate. The source location thus becomes:

$$\xi_s = \frac{1}{\pi} \left(\frac{Q_p}{ud}\right)^2 \tag{10}$$

For a river, the velocity can be estimated as Q_R/Wd where Q_R is the total river flow and W the width. Making the width finite is inconsistent with no reflection at the far boundary, but compatible with the type of simplifying algebraic assumptions employed. The source location becomes:

$$\xi_s = \frac{W^2}{\pi} \left(\frac{Q_p}{Q_R}\right)^2 \tag{11}$$

thus is a simple function of stream width and ratio of plant pumping rate to river flow.

Graphical summary for streams - The two-dimensional distribution relation, Equation 5, can be represented conveniently for a stream width W by defining and substituting a non-dimensional width as:

$$y' = \frac{y}{W} \tag{12}$$

and a non-dimensional downstream coordinate as:

$$\xi' = \frac{\xi}{W^2} = \frac{x\ D_y}{W^2\ u} = \frac{x\ D_y\ d}{WQ_R} \tag{13}$$

and a non-dimensional source location of:

$$\xi_s' = \frac{\xi_s}{W^2} = \frac{1}{\pi}\ (\frac{Q_p}{Q_R})^2 \tag{14}$$

Equation 5 then becomes:

$$\frac{s}{s_p} = (\frac{\xi_s'}{\xi'})^{1/2}\ e^{\frac{-y'^2}{4\xi'}} \tag{15}$$

Since the fully mixed concentration will be:

$$s_m = \frac{Q_p}{Q_R}\ s_p \tag{16}$$

then substituting Equation 14 and 16 into 15 gives the concentration distribution as a ratio to the fully mixed concentration as:

$$\frac{s}{s_m} = \frac{1}{\sqrt{\pi\xi'}} \cdot e^{-y'^2/4\xi'} \qquad (17)$$

which is summarized conveniently in Figure 1 as s/s_m as a function of the non-dimensional downstream coordinate ξ' for different values of the non-dimensional width y'.

It is seen from Figure 1 (and Equation 17) that the concentration ratios converge approximately to a value of 1.0, the fully mixed condition, at a value of the non-dimensional downstream coordinate of;

$$\xi_m' = \frac{1}{\pi} \qquad (18)$$

The distance from the source location to the fully mixed concentration contour is found using Equations 14 and 18 as:

$$L_m = \frac{WQ_R}{dD_y}\left[\xi_m' - \xi_s'\right] = \frac{WQ_R}{dD_y\pi}\left[1 - \left(\frac{Q_P}{Q_R}\right)^2\right] \qquad (19)$$

It should be noted that Figure 1 and the supporting equations assume no reflection at the far stream boundary. If reflections were included in the analysis it would be found that the concentrations for each width, y', would converge more rapidly with downstream position ξ', the relationship for the source position, Equation 14 would become more complex, and the distance to complete mixing would be shorter than that given by Equation 18.

Area within a concentration contour - The lateral concentration distribution, Equation 5, and its source location ξ_s, could be used as a set

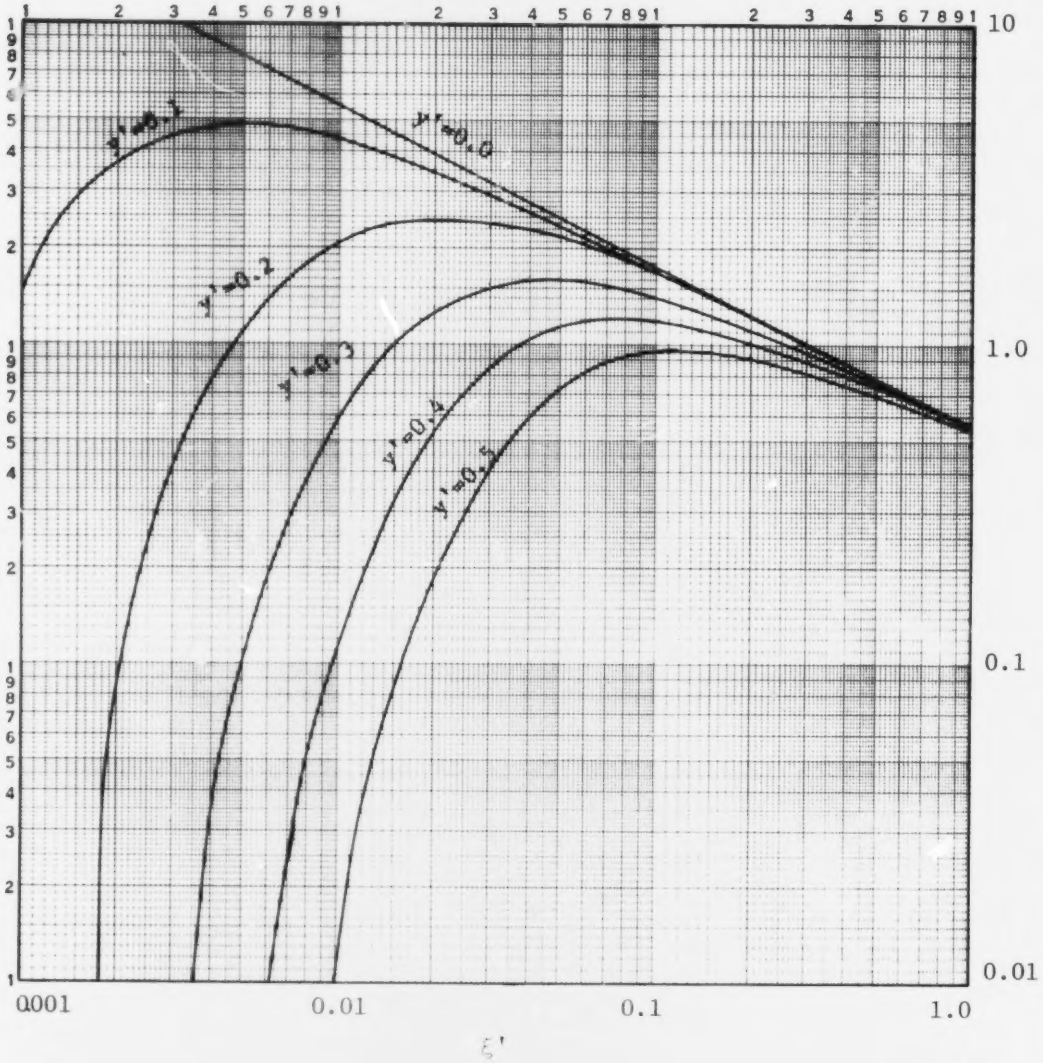

FIGURE 1: CONCENTRATION RATIO TO FULLY MIXED CONCENTRATION AS A FUNCTION OF THE NON-DIMENSIONAL DOWNSTREAM COORDINATE FOR SELECTED VALUES OF THE NON-DIMENSIONAL WIDTH

of relations for comparison to measured lateral concentrations at a number of downstream cross-sections. This requires obtaining a best fit not only within a cross-section but between cross-sections. It also requires adjustment of the coordinates to compensate for cross-stream velocities and initial bending of a plume. One method of circumventing these problems is to consider a more gross algebraic description of concentration distribution. The surface area contained within a given concentration contour provides such a description.

The equation of an iso-concentration curve is, from Equation 5:

$$y = 2\sqrt{\xi}\left\{ \ln\left[\frac{s_p}{s_c}\left(\frac{\xi_s}{\xi}\right)^{1/2}\right]\right\}^{1/2} \qquad (20)$$

where s_c is the specified iso-concentration value. The surface area within this iso-concentration curve then becomes:

$$A = \int_{x_s}^{x_c} y\,dx = \frac{2u}{D_y}\int_{\xi_s}^{\xi_c} \xi^{1/2}\left\{ \ln\left[\frac{s_p}{s_c}\left(\frac{\xi_s}{\xi}\right)^{1/2}\right]\right\}^{1/2} d\xi \qquad (21)$$

where the substitution $dx = \frac{u}{D_y}d\xi$ has been made from Equation 4; x_s (or ξ_s) is the source location and x_c (or ξ_c) is the longitudinal position where the iso-concentration curve intersects $y = 0$. The coordinate position ξ_c is evaluated from Equation 8 to be:

$$\xi_c = \xi_s\left(\frac{s_p}{s_c}\right)^2 \qquad (22)$$

The integral in Equation 21 is made non-dimensional and dependent only on the ratio s_p/s_c by:

$$\eta = \frac{s_c}{s_p} \left(\frac{\xi}{\xi_s}\right)^{1/2} \tag{23}$$

hence, in Equation 21 $d\xi = 2\xi_s \left(\frac{s_p}{s_c}\right)^2 \eta d\eta$; ξ_s becomes $\eta_s = s_c/s_p$, and ξ_c becomes $\eta_c = 1$.

The surface area within an iso-concentration contour then becomes:

$$A = \frac{4u}{D_y} \xi_s^{3/2} \left(\frac{s_c}{s_p}\right)^{-3} \int_{s_c/s_p}^{1} \eta^2 \left[\ln \left(\frac{1}{\eta}\right)\right]^{1/2} d\eta \tag{24}$$

or, using Equation 10, then:

$$A = A_n \left(\frac{s_c}{s_p}\right)^{-3} \cdot \int_{s_c/s_p}^{1} \eta^2 \left[\ln \left(\frac{1}{\eta}\right)\right]^{1/2} d\eta \tag{25}$$

where A_n is a scaling area given as:

$$A_n = \frac{4}{\pi^{3/2}} \cdot \frac{Q_p^3}{D_y u^2 d^3} \tag{26}$$

When the velocity is expressed as for a river flow the scaling area becomes:

$$A_n = \frac{4}{\pi^{3/2}} \frac{W^2 Q_p^3}{D_y Q_R^2 d} \tag{27}$$

For values of s_c/s_p less than about 0.60 the integral in Equations 25 and 26 converges rapidly to approach the constant value of 0.168 hence the relations can be approximated as:

$$\frac{A}{A_n} \simeq 0.168 \left(\frac{s_c}{s_p}\right)^{-3} \tag{28}$$

Application of Figure 2, hence Equation 25 and 26, to field data is relatively simple. Surface temperature observations are plotted on a map and contoured. The area within each temperature contour is determined and the actual areas are plotted against s_c/s_p on log-log paper with the same grid size as Figure 2. The data plot is superimposed on Figure 2 with ordinates of s_c/s_p matched and moved along the abscissa until a good fit can be found between the data plot and theoretical line. The ratio of the actual area to non-dimensional area will be a constant for every point along the abscissa, and the constant will be the area scale given by Equation 26 and 27.

Non-conservative Case

Atmospheric cooling can become important when mixing of a heated discharge takes place over a large surface area. The significance of this term can be determined by examining the two-dimensional case with surface cooling included and comparing it to the conservative case.

The steady-state heat budget for a vertically mixed column cooling to the atmosphere and dispersing laterally into a current can be written as:

$$u \frac{\partial \Theta}{\partial x} = \frac{\partial}{\partial y} \left(D_y \frac{\partial \Theta}{\partial y}\right) - \frac{K}{\rho c_p d} \Theta \tag{29}$$

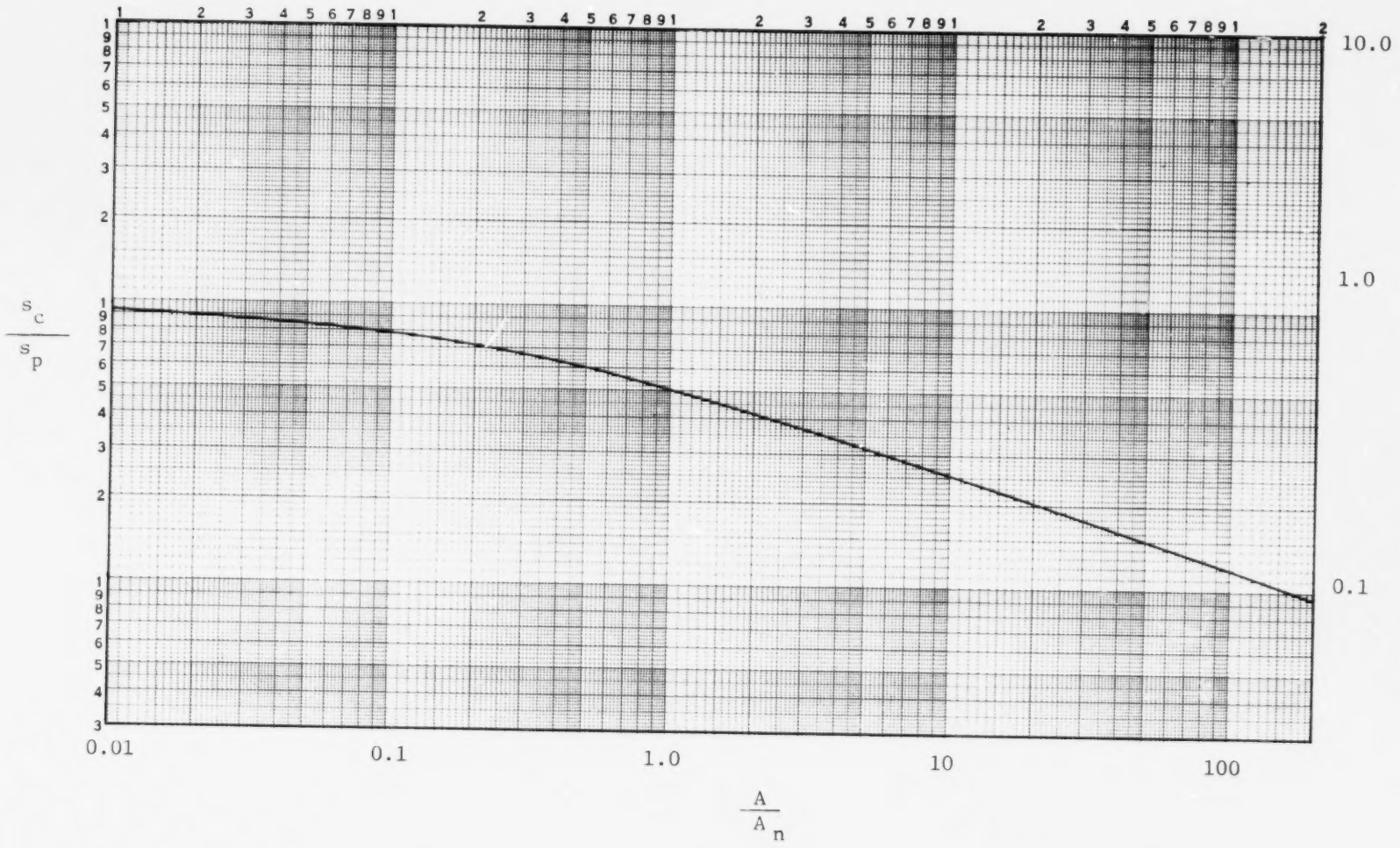

FIGURE 2: NON-DIMENSIONAL SURFACE AREA RATIO AS A FUNCTION OF CONCENTRATION RATIO FOR THE TWO DIMENSIONAL CASE

where Θ is the temperature rise; K is the coefficient of surface heat exchange; d is the depth of water column; and, ρ and c_p are the specific weight and specific heat of water.

Analytic development - Brooks (1960) has examined an expression similar to Equation 29 for the dispersion and die-away of coliforms from a sewage outfall. He showed that a solution could be developed for constant D_y from a transform as:

$$\Theta = \left[s(\xi, y) \right] e^{-\alpha\xi} \tag{30}$$

where

$$\alpha = \frac{K}{\rho c_p D_y d} \tag{31}$$

where $\xi = ux/D_y$ as in Equation 4, and where $s(\xi,y)$ is a solution to:

$$\frac{\partial s}{\partial \xi} = \frac{\partial^2 s}{\partial y^2} \tag{32}$$

which is the two-dimensional conservative case considered previously. An analytic form of the heat balance therefore is:

$$\Theta(\xi, y) = \frac{C}{2\sqrt{\xi}} \cdot e^{-\frac{y^2}{4\xi}} \cdot e^{-\alpha\xi} \tag{33}$$

The constant of integration, C, is evaluated at the source position where $\xi = \xi_s$, $y = 0$, $\Theta = \Theta_p$ giving:

$$\frac{\Theta(\xi, y)}{\Theta_p} = \left(\frac{\xi_s}{\xi}\right)^{1/2} \cdot e^{-\frac{y^2}{4\xi}} \cdot e^{-\alpha(\xi - \xi_s)} \qquad (34)$$

The source coordinate position, ξ_s, is evaluated as previously by equating the source strength, **S**, with advection by the current u giving:

$$\xi_s = \frac{1}{\pi}\left(\frac{S}{ud\Theta_p}\right)^2 \qquad (35)$$

Application to streams - The detailed conservative spatial distribution given in Figure 1 can be employed to compute the non-conservative distribution by first determining the conservative distribution $s(\xi',y')$ and then modifying each value as:

$$\Theta(\xi',y') = s(\xi',y') \cdot e^{-\alpha'(\xi' - \xi_s')} \qquad (36)$$

where ξ_s' and y' have the same definition given previously and:

$$\alpha' = \frac{KW^2}{\rho c_p D_y d} \qquad (37)$$

The mixed temperature rise, Θ_m, when surface cooling is included will be less than the mixed temperature rise, s_m, computed from the ratio of plant pumping rate to stream flow, or:

$$\Theta_m = s_m e^{-\alpha'(\xi_m' - \xi_s')} \qquad (38)$$

where the non-dimensional coordinate position for complete mixing, $\xi_m = 1/\pi$, is the same as for the conservative distribution case. The order of magnitude of the decrease can be estimated based on values of K = 150 Btu $ft^{-2}Day^{-1}{}^{\circ}F^{-1}$, W = 500 ft, d = 10 ft, $D_y = 10^5 ft^2 Day^{-1}$ and $(\xi_m' - \xi_s') = 0.30$. This geometry gives $\Theta_m = 0.20\ s_m$, which is a significant reduction in final mixed temperature due to surface cooling.

Area within a concentration contour - Surface cooling will modify the surface area within a temperature contour. The equation of an iso-concentration contour becomes:

$$y = 2\xi^{1/2}\left\{\ln\left[\left(\frac{\Theta_p}{\Theta_c}\right)\left(\frac{\xi_s}{\xi}\right)^{1/2}\right] - \alpha(\xi-\xi_s)\right\}^{1/2} \qquad (39)$$

where Θ_c is the specified iso-concentration value. The surface area within this is:

$$A = \int_{x_s}^{x_c} y\,dx = \frac{2u}{D_y}\int_{\xi_s}^{\xi_c} \xi^{1/2}\left\{\ln\left[\left(\frac{\Theta_p}{\Theta_c}\right)\left(\frac{\xi_s}{\xi}\right)^{1/2}\right] - \alpha(\xi-\xi_s)\right\}^{1/2} d\xi \qquad (40)$$

where the substitution $dx = \frac{u}{D_y}\,d\xi$ has been made from Equation 4; x_s (or ξ_s) is the source location; and x_s (or ξ_c) is the longitudinal position where the iso-concentration curve intersects y = 0. The limit ξ_c is evaluated from Equation 34 to be a solution to:

$$\sqrt{\xi_c}\cdot e^{2\xi_c} = \frac{\Theta_p}{\Theta_c}\sqrt{\xi_s}\cdot e^{\alpha\xi_s} \qquad (41)$$

and where ξ_s is known from Equation 35.

The integral in Equation 40 is made non-dimensional and dependent on the ratio Θ_p/Θ_c by:

$$\eta = \frac{\Theta_c}{\Theta_p} \left(\frac{\xi}{\xi_s}\right)^{1/2} \tag{42}$$

hence, $d\xi = 2\xi_s(\Theta_p/\Theta_c)^2\eta^2 d\eta$; the limit ξ_s becomes $\eta_s = \Theta_c/\Theta_p$ and η_c is evaluated from Equation 41. The surface area within an iso-concentration contour becomes:

$$\frac{A}{A_n} = \left(\frac{\Theta_c}{\Theta_p}\right)^{-3} \int_{\Theta_c/\Theta_p}^{\eta_c} \eta^2 \left\{ \ln\left(\frac{1}{\eta}\right) - \beta \left[\left(\frac{\Theta_p}{\Theta_c}\right)\eta^2 - 1 \right] \right\}^{1/2} d\eta \tag{43}$$

where β is defined as:

$$\beta = \alpha\xi_s = \frac{1}{\pi} \frac{K Q_p^2}{\rho c_p D_y d^3 u^2} \tag{44}$$

and the scaling area, A_n, is defined as previously to be:

$$A_n = \frac{4}{\pi^{3/2}} \cdot \frac{Q_p^3}{D_y u^2 d^3} \tag{45}$$

When the mean velocity is evaluated as $u = Q_R/Wd$ as for a stream, then:

$$\beta = \frac{1}{\pi} \frac{K W^2}{\rho c_p D_y d} \left(\frac{Q_p}{Q_R}\right)^2 \tag{46}$$

and,

$$A_n = \frac{4}{\pi^{3/2}} \cdot \frac{W^2}{d} \cdot \frac{Q_p}{D_y} \left(\frac{Q_p}{Q_R}\right)^2 \tag{47}$$

The magnitude of the decay term, β, estimated for values of K = 150 Btu $ft^{-2}Day^{-1}{}^{o}F^{-1}$, W = 500 ft, d = 10' $D_y = 10^5 ft^2 Day^{-1}$, and $Q_p/Q_R = 0.3$ is, from Equation 46, $\beta = 0.01$ and since it increases with the square of the stream width can be as large as 0.5.

The non-dimensional integral in Equation 43 has been evaluated numerically for selected values of β and the result is given in Figure 3 where Θ_c/Θ_p is given as a function of A/A_n for selected values of β. For $\beta = 0$ this gives the conservative case as it should. It is seen that cooling has little influence until Θ_c/Θ_p reaches 0.60 by mixing. However, cooling changes the slope of the Θ_c/Θ_p vs. A/A_n relation. Also from Equation 44 and 45 or 46 and 47 it is seen that:

$$\beta = \frac{\sqrt{\pi}}{4} \cdot \frac{K A_n}{\rho c_p Q_p} \qquad (48)$$

which is independent of D_y. This implies that if K and Q_p are known when data is matched to the theoretical curves in Figure 3, the value of A_n found from matching the abscissas must be compatible with the value of β on the curve along which the observations fall.

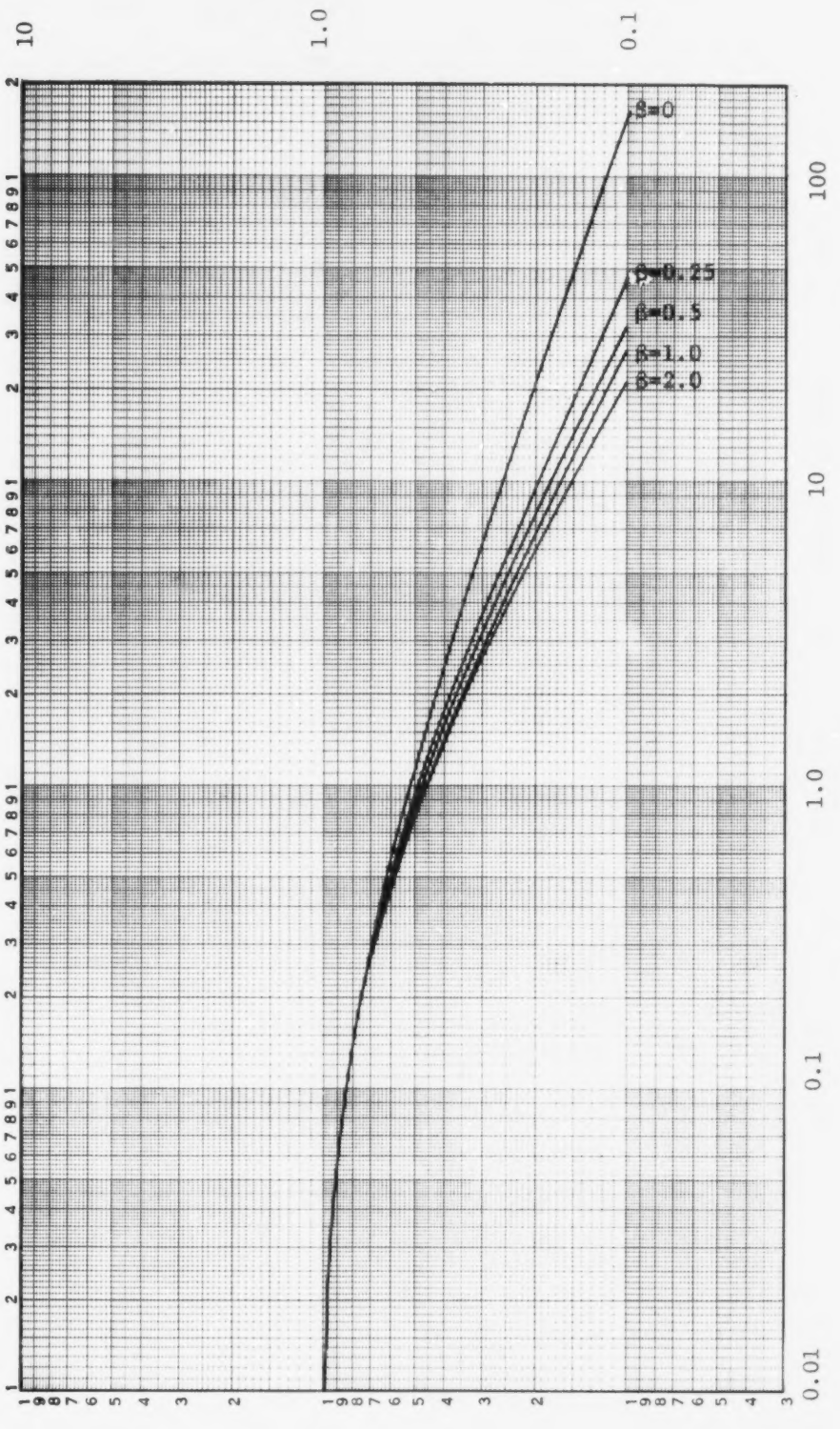

FIGURE 3: RELATIONSHIP OF TEMPERATURE RISE RATIO TO NON-DIMENSIONAL SURFACE AREA RATIO FOR SELECTED VALUES OF β

THE THREE-DIMENSIONAL CASE

The next degree of spatial complexity beyond the two dimensional case is to consider the lateral and vertical dispersion of heat from a point source into a longitudinal velocity field. The balance for a conservative substance becomes:

$$\frac{u\partial s}{\partial x} = \frac{\partial}{\partial y}\left(D_y \frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial s}{\partial z}\right) \tag{49}$$

For constant u, D_y, and D_z, Equation 49 has the analytic form of:

$$s(x,y,z) = \frac{C}{x} \exp\left[-\frac{1}{4x}\left(\frac{y^2}{\sigma_y} + \frac{z^2}{\sigma_z}\right)\right] \tag{50}$$

where C is a constant of integration and:

$$\sigma_y = \frac{D_y}{u} \quad , \quad \sigma_z = \frac{D_z}{u} \tag{51}$$

The constant of integration is evaluated at the source position $x = x_s$, $y = 0$, $z = 0$ where $s = s_p$, the source concentration. This gives:

$$\frac{s(x,y,z)}{s_p} = \left(\frac{x_s}{x}\right) \cdot \exp\left[-\frac{1}{4x}\left(\frac{y^2}{\sigma_y} + \frac{z^2}{\sigma_z}\right)\right] \tag{52}$$

The source position, x_s, is evaluated by equating the source strength, S, to advection in the mean velocity field as:

$$S = u \int_0^\infty \int_0^\infty s(x,y,z)\,dydz \tag{53}$$

which requires integrating:

$$S = ux_s s_p \int_0^{\infty} \int_0^{\infty} \frac{1}{x} \cdot \exp\left[- \frac{1}{4x} \left(\frac{y^2}{\sigma_y} + \frac{z^2}{\sigma_z}\right) \right] dydz \qquad (54)$$

The double integral is a definite integral independent of x and has a value of $\pi\sqrt{\sigma_y \sigma_z}$.

The source coordinate position becomes:

$$x_s = \frac{(S/s_p)}{\pi\sqrt{D_y D_z}} = \frac{Q_p}{\pi\sqrt{D_y D_z}} \qquad (55)$$

It is independent of the uniform current, u, and depends on the product of the lateral and vertical diffusivities.

Surface Area Within a Concentration Contour

The equation of a specific iso-concentration contour, s_c, on the surface, z = 0, is found from Equation 52 to be:

$$y = 2\sqrt{\sigma_y}\ x^{1/2} \left\{ \ln \left[\left(\frac{s_p}{s_c}\right) \left(\frac{x_s}{x}\right) \right] \right\}^{1/2} \qquad (56)$$

and the surface area within an iso-concentration contour is given by:

$$A = \int_{x_s}^{x_c} y dx = 2\sqrt{\sigma_y} \int_{x_s}^{x_c} x^{1/2} \left\{ \ln \left[\left(\frac{s_p}{s_c}\right) \left(\frac{x_s}{x}\right) \right] \right\}^{1/2} dx \qquad (57)$$

where x_c is found from Equation 52 for y = 0, z = 0 to be:

$$x_c = \left(\frac{s_p}{s_c}\right) x_s \qquad (58)$$

The integral in Equation 57 can be simplified by letting $\eta = \frac{s_c}{s_p} \cdot \frac{x}{x_s}$ and the limit x_s becomes $\eta_s = (Sc/Sp)$ and X_c becomes $\eta_c = 1$. This gives:

$$\frac{A}{A_n} = \left(\frac{s_c}{s_p}\right)^{-3/2} \int_{s_c/s_p}^{1} \eta^{1/2} \left[\ln\left(\frac{1}{\eta}\right)\right]^{1/2} d\eta \qquad (59)$$

where A_n is a scaling area given by:

$$A_n = 2\sqrt{\sigma_y}\ x_s^{3/2} \qquad (60)$$

and substituting from Equation 55 for x_s gives:

$$A_n = \frac{Q_P\sqrt{Wd}}{\sqrt{2}\,\pi^{3/2}\,(D_y D_z^{\,3})^{1/4}} \left(\frac{Q_P}{Q_R}\right)^{1/2} \qquad (61)$$

The concentration ratio, s_c/s_p, is shown as a function of the area ratio, A/A_n, in Figure 4. Comparison with the two-dimensional conservative case also shown on Figure 4 shows that area increases inversely with the 3/2 power of the concentration ratio as compared with the cubic power for the two-dimensional case. Also the three-dimensional scaling area increases with the 3/2 power of the plant pumping rate as compared with the cubic power for the two-dimensional case.

For values of s_c/s_p less than 0.60, the integral in equation converges rapidly to approach the constant value of 0.496, hence the relationship can be approximated as:

$$\frac{A}{A_n} \simeq 0.496 \left(\frac{s_c}{s_p}\right)^{-3/2} \qquad (62)$$

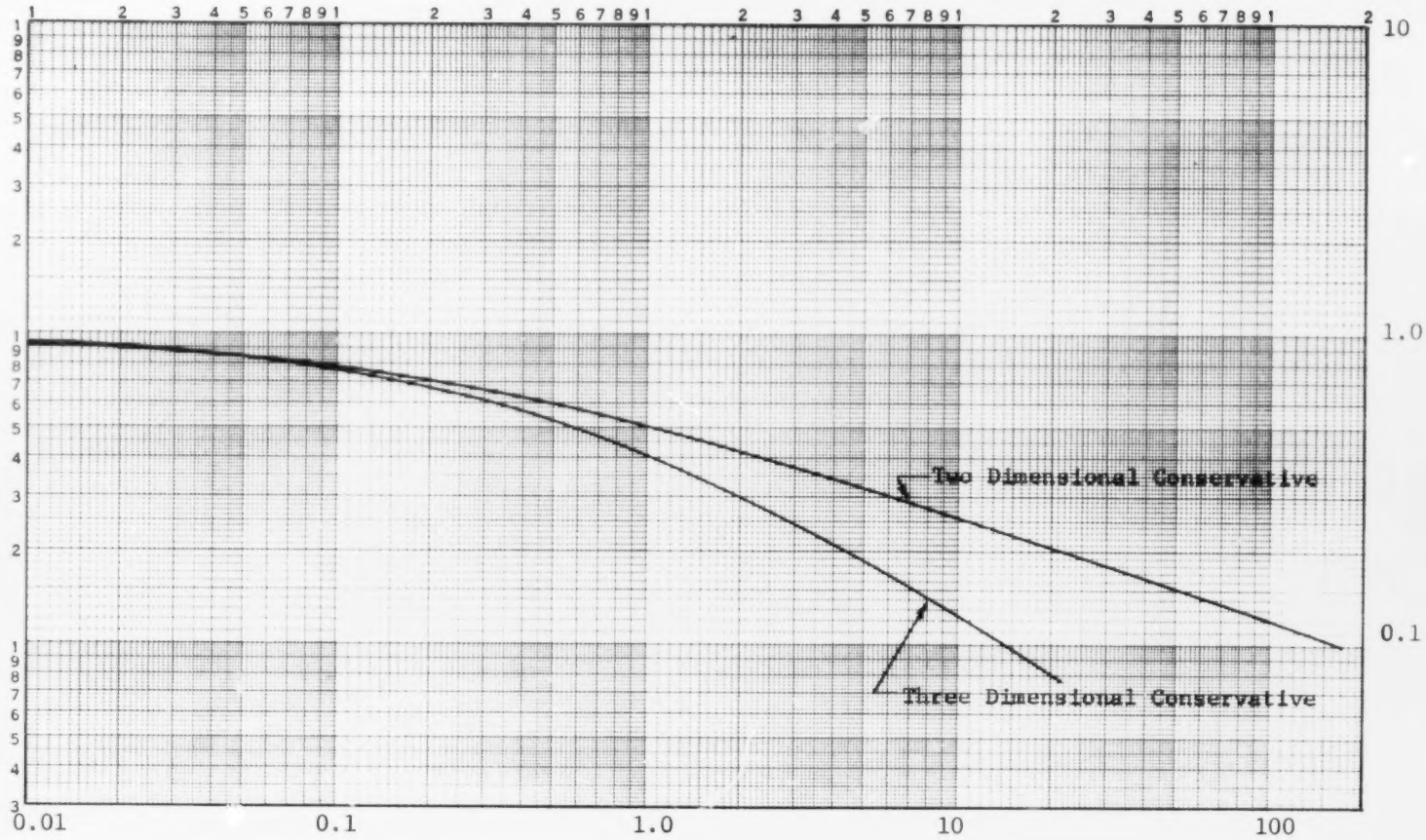

FIGURE 4: NON-DIMENSIONAL SURFACE AREA RATIO AS A FUNCTION OF CONCENTRATION RATIO FOR THE TWO DIMENSIONAL AND THREE DIMENSIONAL CASES

Cross-sectional Area Within Iso-concentration Contour

It appears more practical to analyze a three-dimensional temperature distribution from cross-sectional data than from surface area data. For prediction purposes it appears useful to have methods for estimating the fraction of cross-sectional area contained within an iso-concentration contour as a function of downstream coordinate position.

The cross-sectional area within an iso-concentration contour at a downstream coordinate position, x_c, is found as:

$$A_c = \int_0^{y_c} z\,dy \tag{63}$$

where $y = y_c$ is found for the particular iso-concentration value, s_c, and is a function of downstream coordinate position. Equation 52 gives the depth, z, to the iso-concentration value, s_c, as:

$$z = \left\{ 4x\sigma_z \ln \left[\left(\frac{s_p}{s_c}\right) \left(\frac{x_s}{x_c}\right) \right] - \frac{\sigma_z}{\sigma_y} y^2 \right\}^{1/2} \tag{64}$$

Inserting Equation 64 into Equation 63 gives:

$$A_c = \left(\frac{\sigma_z}{\sigma_y}\right)^{1/2} \int_0^{y_c} \left[F^2(x) - y^2 \right] dy \tag{65}$$

where:

$$F^2(x) = 4x\sigma_y \ln \left[\left(\frac{s_p}{s_c}\right) \left(\frac{x_s}{x}\right) \right] \tag{66}$$

Equation 65 integrates to:

$$A_c = \frac{1}{2}\left(\frac{\sigma_z}{\sigma_y}\right)^2 \left[y_c\left(F^2(x) - y_c^2\right)^{1/2} + F^2(x)\, \mathrm{Sin}^{-1}\left(\frac{y_c}{F(x)}\right)\right] \quad (67)$$

Since $y = y_c$ at $z = 0$, it is found from Equation 64 that

$$y_c^2 = 4x\sigma_y \ln\left[\left(\frac{s_p}{s_c}\right)\left(\frac{x_s}{x}\right)\right] = F^2(x) \quad (68)$$

thus from Equation 67, it is found that:

$$A_c = \frac{\pi}{4}\left(\frac{\sigma_z}{\sigma_y}\right)^{1/2} F^2(x) = (\sigma_z\sigma_y)^{1/2}\, \pi x \ln\left[\left(\frac{s_p}{s_c}\right)\left(\frac{x_s}{x}\right)\right] \quad (69)$$

which has no convenient interpretation. Equation 69 can be rearranged to:

$$\frac{s_c}{s_p} = \frac{x_s}{x} \exp\left[-\frac{u\, A_c}{\pi x\sqrt{D_y D_z}}\right] \quad (70)$$

and for a river flow using $u = Q_R/A_R$, where A_R is the total cross-sectional area of the river, and using the value of x_s from Equation 56, the above expression simplifies to:

$$\frac{s_c}{s_p} = f(x) \exp\left[-g(x)\frac{A_c}{A_R}\right] \quad (71)$$

or a fraction of the cross-sectional area within a contour, may be expressed as:

$$\frac{A_c}{A_R} = \frac{1}{g(x)} \ln\left[\left(\frac{s_p}{s_c}\right) f(x)\right] \quad (72)$$

where:

$$f(x) = \frac{Q_p}{\pi x\sqrt{D_y D_z}} \quad ; \quad g(x) = \frac{Q_R}{\pi x\sqrt{D_y D_z}} \tag{73}$$

and:

$$f(x)/g(x) = Q_p/Q_R \tag{74}$$

Equation 71 suggests that at each cross-section s_c/s_p will be log- linear. with A_c/A_R for that cross-section and slope of the relation will be g(x) while the intercept will be f(x).

Since the measured distance from the discharge to the cross-section will be:

$$L = x - x_s \tag{75}$$

then g(x) will follow the relationship:

$$\frac{1}{g(x)} = \frac{Q_p}{Q_R} + \frac{\pi\sqrt{D_y D_z}}{Q_R} L \tag{76}$$

which is a simple linear function of distance, L, from the discharge to the cross-section. Also, f(x), the intercept, will follow the relation:

$$\frac{1}{f(x)} = 1 + \frac{\pi\sqrt{D_y D_z}}{Q_p} L \tag{77}$$

This suggests the following analysis of cross-sectional data:

1. Plot the cross-section temperature distribution, contour s_c, and find A_c/A_r for each s_c contour.

2. Plot A_c/A_r vs. s_c/s_p for each cross-section and determine g(x) at that cross-section.

3. After doing the above for each cross-section, plot $1/g(x)$ vs. distance, L, from the discharge to the cross-section for comparison with Equation 76.

4. A. similar analysis can be made of the log-linear intercept, f(x).

For a river, the condition for complete mixing can be approximated by $s_c = s_m$ where for a conservative substance:

$$s_m = \frac{s_p Q_p}{Q_R} \tag{78}$$

and from Equation 71:

$$\frac{1}{f(x_m)} = \frac{s_p}{s_m} = \frac{Q_R}{Q_p} \tag{79}$$

at x_m and substituting in Equation 77 gives the longitudinal distance from the discharge to the completely mixed contour as:

$$L_m = \frac{Q_R}{\pi\sqrt{D_y D_z}} \cdot \left(1 - \frac{Q_p}{Q_R}\right) \tag{80}$$

SUMMARY OF PROPERTIES FOR INITIAL TEMPERATURE DISTRIBUTIONS FROM SIMPLE HEAT BALANCES

The study of dispersion into a uniform current as presented here is but an initial analysis of the complex problem of initial dispersion of a heated discharge. The sequence of formulae provide a number of geometric laws which at least satisfy a heat balance during initial mixing. They are dimensionally correct and show interrelations of the basic parameters of waterbody velocity and dimensions such as width and depth, plant pumping rate and source concentration. The formulae have been rearranged in a form which have parameters grouped as ratios and multipliers and suggest possible empirical relations for correlation to field data. The derived relations provide a number of simple deductions about initial dispersion which will be summarized first for the two dimensional case, then for the three-dimensional case.

Properties of Two Dimensional Distributions

The significant properties of two dimensional distributions which arise from this analysis are:

1. The surface area varies inversely with the cubic power of the concentration ratio. Other cases (inclusion of surface cooling, three-dimensional distribution) will have a different power law. Applicability of the two-dimensional conservative case can therefore be judged by comparison of the slopes of the theoretical and observed data plots.

2. For a given concentration ratio, the area varies with the cubic power of the plant pumping rate. This provides a very simple scaling rule for projecting observations made at one plant size to predictions for an increased plant size. It also indicates that variation in plant pumping rate is an important factor when choosing field test conditions.

3. For a given concentration ratio, the area varies universally with the square of river flow. The smaller the river flow, the larger the surface area required to achieve the same degree of mixing.

4. The distance from the discharge to the fully mixed contour for a discharge into a river flow is:

$$L_m = \frac{WQ_R}{\pi D_y d}\left[1 - \left(\frac{Q_p}{Q_R}\right)^2\right] \qquad (81)$$

and is a function of the ratio of plant pumping rate to river flow as well as the river flow itself.

5. The one parameter entering the relations which is difficult to evaluate is the lateral dispersion coefficient, D_y. First testing an observed distribution for fit to the two-dimensional case, then estimating values for D_y will be a necessary objective of field experimentation.

6. Surface cooling can significantly reduce the area over which mixing will take place. In the case of rivers it results in a lower fully mixed temperature rise which can be computed from Equation 38.

7. Surface cooling changes the slope of the temperature rise-area relation as can be seen from Figure 3.

Properties of Three-Dimensional Distributions

The significant properties of three-dimensional distributions which arises from this analysis are:

1. The surface area within a concentration contour varies inversely with the 3/2 power of the concentration ratio. This is a more rapid decrease than for the two-dimensional non-conservative distribution at the largest estimated die-away as can be seen from Figure 4.

2. The distance from the discharge to the completely mixed contour is given by:

$$L_m = \frac{Q_R}{\pi\sqrt{D_y D_z}}\left(1 - \frac{Q_p}{Q_R}\right) \qquad (82)$$

and varies linearly with the ratio of plant pumping rate to river flow as compared to the square of the ratio for the two-dimensional case.

4. The three-dimensional case can be described by the fraction of cross-sectional area within a concentration contour as a function of distance from the discharge. At a given cross-section location the concentration ratio is a log-linear function of location of cross-sectional area. The inverse slope of the log-linear relation is a simple linear function of distance from the discharge.

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CASE COMPARISONS WITH FIELD DATA

A number of properties of the derived relations can be illustrated by comparison to field data. This is a necessary step in the analysis to determine the applicability of the formulae, and in providing a basis for judging the deviation or scatter of actual observations from predicted conditions.

Discharge into Open Water

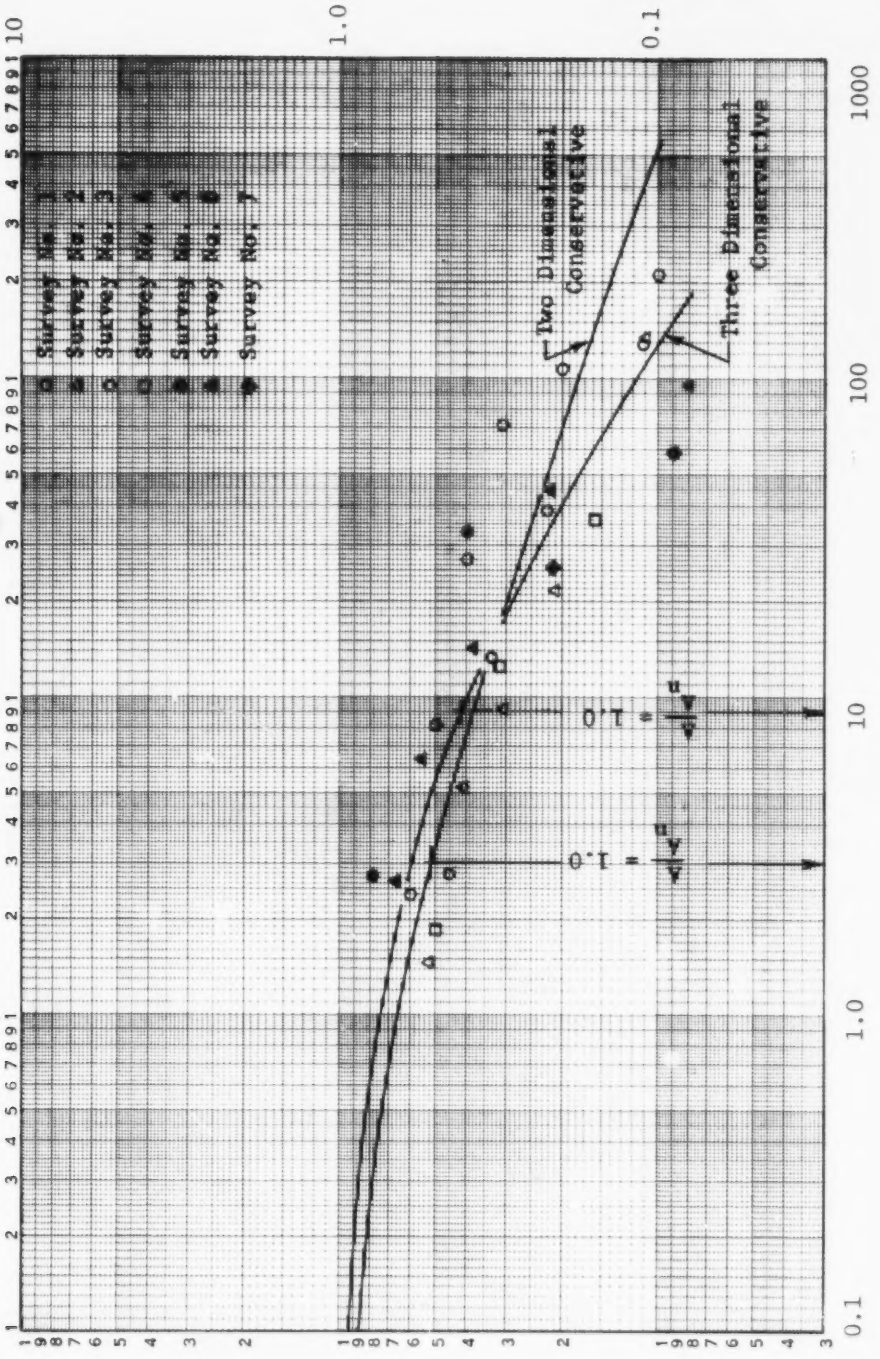
Cheney, et.al. (1965) has reported data resulting from aerial surveys of an 850 MW power plant discharge into an unstratified surface layer in a coastal water. Eight separate sets of data were presented for three different days and seven different power plant loadings. The plant pumping rate was constant for all surveys at 470,000 GPM; but, tidal and weather conditions varied widely between surveys. This data is summarized in Table 1 and plotted in Figure 5 with the two-dimensional and three-dimensional theoretical curves superimposed.

Using surface area data alone, it is seen from Figure 5 that it is difficult to distinguish between applicability of the two-dimensional and three-dimensional cases. The scaling area for the two-dimensional case is 30×10^3 ft^2 and for the three-dimensional case, 90×10^3 ft^2. According to the previous analyses, if plant pumping rate were doubled, the two-dimensional analysis would predict an eight-fold increase in area to reach a given temperature rise; while, the three-dimensional analysis would predict less than a three-fold increase in area. For the present plant size, it is seen from the data that the heat mixed to within one-tenth the condenser rise over an area of 200 acres.

TABLE 1 - SURFACE TEMPERATURE INFLUENCE
MEASURED AT MORRO BAY POWER PLANT, 1963 and 1964

Survey No.	Condenser Rise °F	Contour Rise °F	Area Enclosed $Ft^2 \times 10^{-4}$
1	18	8	2.94
		6	13.42
		4	42.00
		2	126.00
2	19	10	1.45
		8	5.25
		6	9.45
		4	21.20
		2	126.00
3	20	12	2.40
		10	8.57
		8	28.60
		6	71.30
		4	105.00
		2	160.00
4	12	6	1.93
		4	12.60
		2	37.20
5	5	4	2.86
		2	37.20
6	4	2	0.97
7	13	9	2.52
		7	5.88
		5	14.30
		3	46.20
		1	95.40
8	13	3	23.10
		1	56.70

After Cheney, et.al. (1965)



$\frac{S_c}{S_p}$

FIGURE 5: TEMPERATURE RISE RATIO VERSUS SURFACE AREA WITHIN THE TEMPERATURE RISE CONTOUR FOR THE DATA OF CHENEY AND RICHARDS

The influence of cooling can be examined by assuming that the two-dimensional analysis is applicable. For the discharge rate of 470,000 GPM or 98 x 10^6 ft^3 Day^{-1} and assuming K = 150 Btu ft^{-2} Day^{-1} F^{-1} it is seen from Equation 48 that $\beta = 3 \times 10^{-4}$ and the conservative distribution comparison appears warranted for this case.

The surface area data is not sufficiently consistent to estimate values of the non-dimensional area for each individual survey. Use of different plant pumping rates would have made the data more meaningful and might aid in choosing the applicability of one case over another. Even if a variation in the area scale could be detected between surveys, it would be necessary to have other types of field observations to separate out the influence of mean velocity, depth of vertical mixing, and the diffusivities which appear in the scaling area formulae.

River Discharge Analysis

Three recent surveys are available for the TVA Widows Creek Steam Plant, one by TVA (8/30/67) and two by Vanderbilt University (11/20/68), (11/21/68). The data shows the heat spreading laterally and vertically as it moves downstream from the plant. Sufficient data were collected to analyze both the surface area contours and the cross-sectional area contours. Details of the Vanderbilt surveys will be covered in a subsequent report. The surface area temperature data for each survey is given in Table 2 and the cross-sectional area data is summarized in Table 3.

Surface area as a function of temperature rise ratio for each survey is shown in Figure 6. It has been over-laid on the generalized three-dimensional relation of Figure 4 giving the scaling area for each survey shown in Table 2. Comparison of the data plot, Figure 6, with the

TABLE 2 - SURFACE AREA TEMPERATURE DATA

FROM WIDOWS CREEK STEAM PLANT

Survey	River Flow cfs	Plant Flow cfs	Plant Rise, s_p °F	Stream Rise, s_c °F	s_c/s_p	Surface Area$_6$ Ft x 10^6	Scaling Area A_n, $10^6 Ft^2$	$D_y D_z^3$ $Ft^8 Day^{-4}$
Widows Creek, V.U. No.1 11/20/68	26,170	2,370	14.2	1.9	.134	8.20	0.85	4.21×10^{12}
				2.9	.204	5.50		
				3.9	.274	3.66		
				4.9	.345	1.55		
				5.9	.415	.895		
				6.9	.486	.454		
				7.9	.556	.335		
				8.9	.627	.239		
				9.9	.697	.167		
				10.9	.768	.120		
Widows Creek, V.U. No.2 11/21/68	26,170	1,840	13.0	1.9	.146	7.20	0.75	1.70×10^{12}
				2.9	.223	4.16		
				3.9	.300	2.54		
				4.9	.377	1.62		
				5.9	.453	.718		
				6.9	.530	.454		
				7.9	.607	.287		
				8.9	.685	.191		
				9.9	.762	.0955		
Widows Creek, Churchill No.1 8/30/67	47,000	2,200	14.2	1.3	.0915	6.46	0.50	6.20×10^{12}
				2.3	.162	4.58		
				3.3	.232	3.26		
				4.3	.303	2.41		
				5.3	.373	1.27		
				6.3	.434	0.55		
				7.3	.514	0.26		
				8.3	.584	0.15		
				9.3	.655	0.11		

*Section Width, W = 1200 Ft

Section Depth, d = 20 Ft

TABLE 3 - CROSS-SECTIONAL AREA TEMPERATURE

DATA FROM WIDOWS CREEK STEAM PLANT

Survey	River Flow cfs	Plant Flow cfs	Plant Rise s_p	Downstream Distance ft	s_c/s_p 0.7	0.6	0.5	0.4	0.3	0.2	0.1
					A/A_n						
Widows Creek, V.U. No.1 11/20/68	26,170	2,370	14.2	200	.02	.05	.09	.11	.133	.165	.192
				1,525			.006	.023	.043	.116	.219
				2,845					.062	.159	.420
				4,170					.019	.089	.404
				5,490						.182	
Widows Creek, V.U. No.2 11/21/68	26,170	1,840	13.0	200	.013	.020	.060	.086	.096	.130	.155
				1,525			.010	.039	.079	.152	.269
				2,845				.003	.032	.106	.202
				4,170					.061	.172	.492
				5,490						.032	.387
Widows Creek, Churchill No. 1 8/30/67	47,000	2,200	14.2	200	.0184	.0328	.0454	.0561	.0660	.0889	.122
				1,525			.0071	.0237	.0526	.0917	.212
				2,845				.0102	.0300	.0460	.142
				4,170					.0207	.0458	.124
				5,490							.156

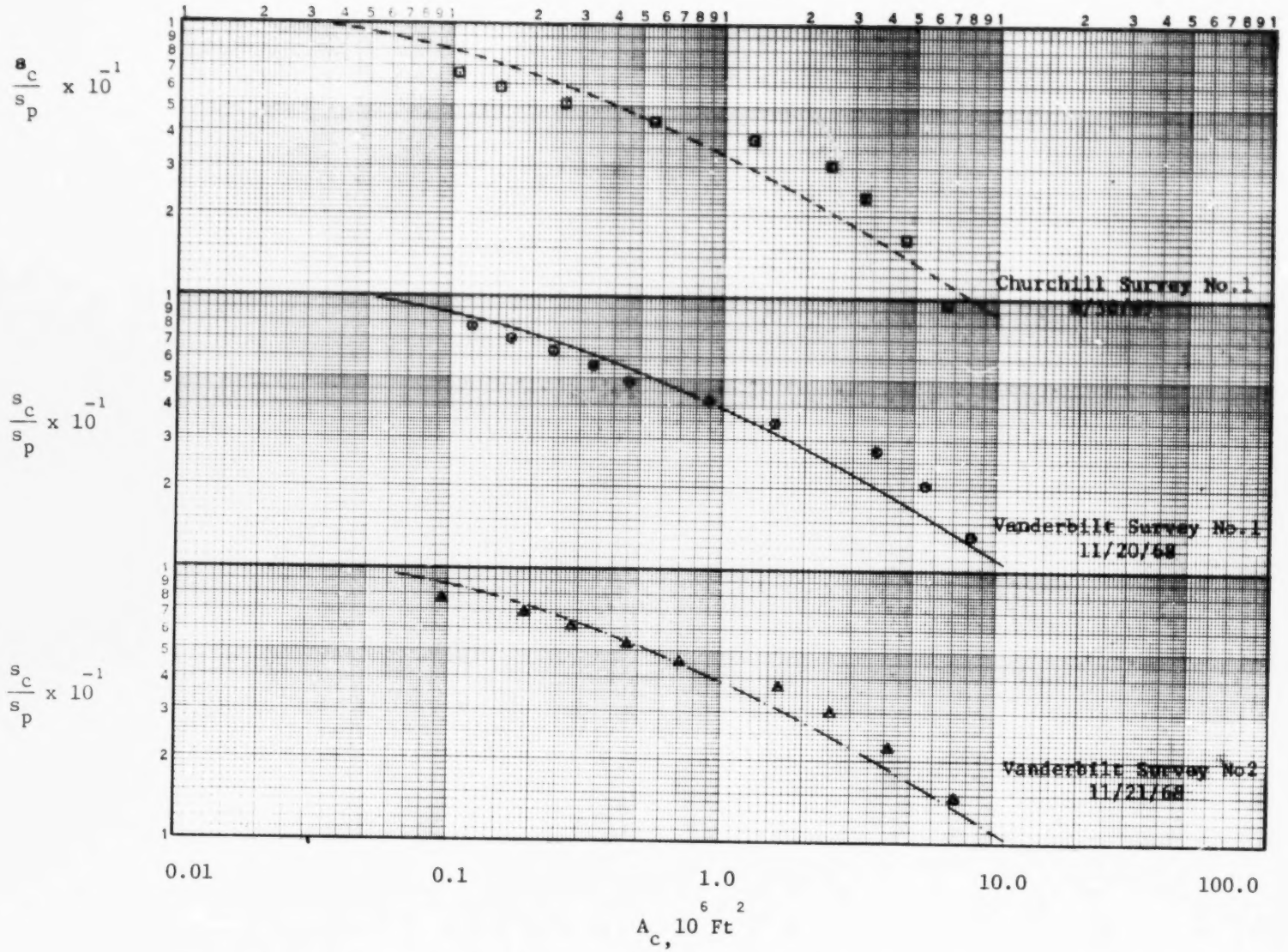


FIGURE 6: TEMPERATURE RISE RATIO VERSUS SURFACE AREA WITHIN THE TEMPERATURE RISE CONTOUR FOR DATA FROM WIDOWS CREEK STEAM PLANT

generalized two-dimensional relations, Figure 2, shows that the three-dimensional curve is definitely the better fit as would be expected.

With the river flow, Q_R, plant flow, Q_p, river width, W, and depth, d, and scaling area A_n known, Equation 61 can be solved for the term $D_y D_z^3$. The values of these diffusivity products are given in Table 4.

The cross-sectional data from Table 3 has been plotted as shown in Figure 7 for the Vanderbilt University No. 1 survey. For each cross-section a semi-log line was fitted using Equation 71 to find g(x)(the log base -e slope) and requiring that the intercept, f(x), follow Equation 74. A least squares fit was used, and the values of f(x) and g(x) found at each cross-section for each survey are summarized in Table 4. Following Equation 76, the values of $1/g(x)$ as a function of distance are shown in Figure 8 for each survey. Since river flows are known, the slope of the lines in Figure 8 can be used to determine the product term $(D_y D_z)$.

Since the term $D_y D_z^3$ is determined from the surface area analysis and the term $D_y D_z$ from the cross-section analysis then D_y and D_z can be separated. The values of D_y and D_z found for each survey are summarized in Table 4.

From this analysis it may be concluded that the derived relationships match the Widows Creek surveys fairly well and were useful in examining and reducing the observed data.

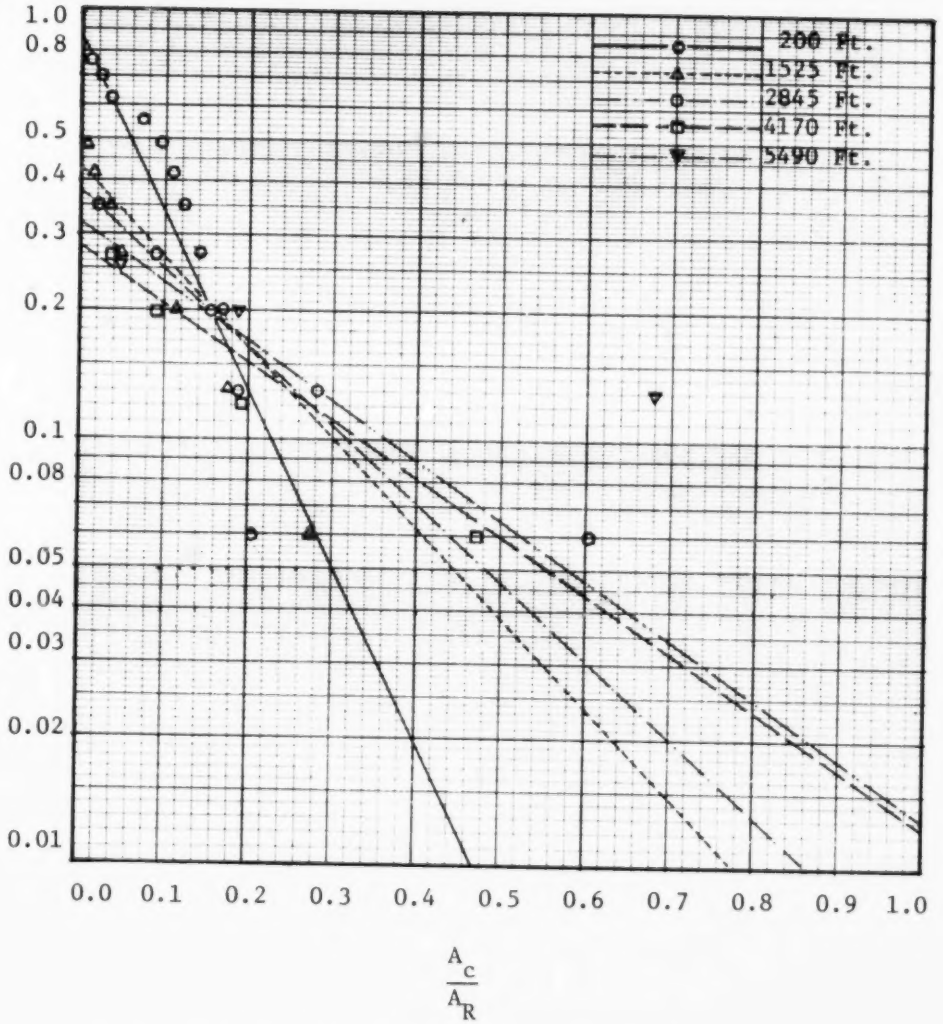


FIGURE 7: RATIO OF TEMPERATURE RISE TO FRACTION OF CROSS-SECTIONAL AREA WITHIN THE TEMPERATURE RISE CONTOUR AT EACH DOWNSTREAM CROSS-SECTION

TABLE 4 - LATERAL AND VERTICAL DISPERSION COEFFICIENTS

DETERMINED FROM WIDOWS CREEK TEMPERATURE DATA

Survey	River Flow	Plant Flow	Downstream Distance	Intercept f(x)	Slope g(x)	$D_y D_z$	D_y	D_z
	cfs	cfs	ft		(base e)	$Ft^4 Day^{-2}$	Ft^2/Day	Ft^2/Day
Widows Creek, V.U. No.1 11/20/68	26,170	2,370	200	.862	9.57	1.39×10^9	25.3×10^6	55.0
			1,525	.439	4.85			
			2,845	.382	4.22			
			4,170	.283	3.13			
			5,490	.321	3.45			
Widows Creek, V.U. No.2 11/21/68	26,170	1,840	200	.822	11.7	1.10×10^9	28.2×10^6	39.2
			1,525	.536	7.62			
			2,845	.373	5.31			
			4,170	.449	6.38			
			5,490	.209	2.98			
Widows Creek, Churchill No. 1 8/30/67	47,000	2,200	200	1.00	21.38	1.26×10^9	18.1×10^6	70.1
			1,525	.528	11.27			
			2,845	.403	8.6			
			4,170	.304	6.49			

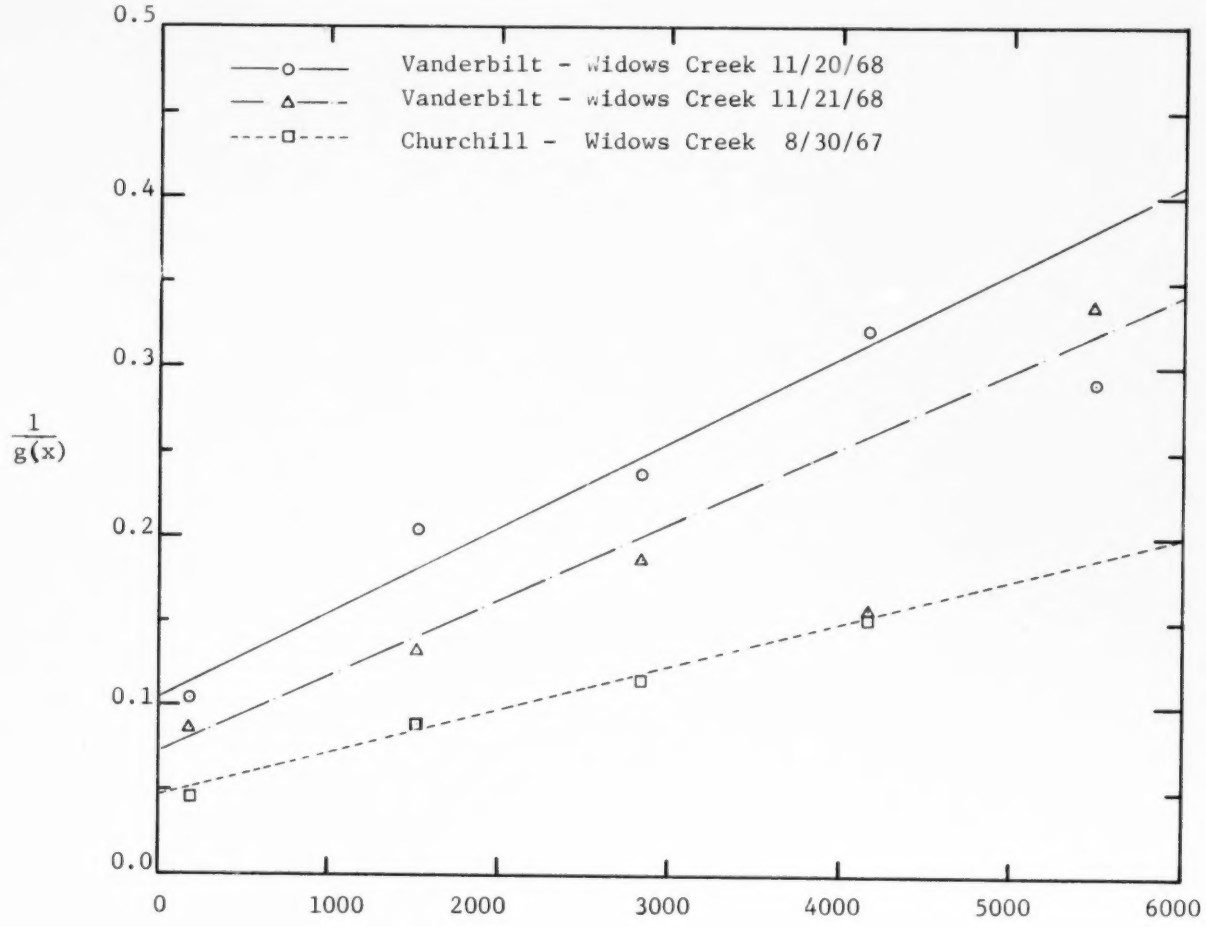


FIGURE 8: RELATIONSHIP OF $\frac{1}{g(x)}$ AND DISTANCE DOWNSTREAM, L, BELOW WIDOWS CREEK STEAM PLANT

ESTIMATING DIFFUSIVITIES

Values of the diffusivities, D_y and D_z, required to evaluate the various formulae are virtually unknown for conditions near power plant discharges. Often these are determined empirically from field data; however, they should be used only in the particular formulae from which they are derived when treated as a single unknown. For example, a lateral diffusivity determined from data by applying the two-dimensional case will have a different value from that determined by applying the same data to the three-dimensional formulae. Also, items not considered in the simple formulae such as initial momentum of the discharge, reflection [illegible]nks and bottoms, and angle of the plume or concentration distributi[illegible]ve to the source bank will also influence the values of empirically derived dispersion coefficients.

The derived formulae have been arranged so that diffusivities appear as simple multipliers of the major temperature distribution properties being described. As a first approximation the diffusivities can be conceived of as scaling parameters related to the distances over which initial dispersion is taking place. It has been suggested by Brooks (1960) that diffusivities can be estimated from the well known 4/3 scaling law, as:

$$D = 86\, L_s^{4/3}$$

where D is the diffusivity ($ft^2 Day^{-1}$) for dispersion for some scale length, L_s (ft), over which dispersion is taking place.

For the case of dispersion in rivers, Parker (1961) and Krenkel (1962) have suggested that the scale can be related to the river geometry. If, for the river case examined previously, it is assumed that the lateral

diffusivity is related to the width, W, and the vertical diffusivity to the depth, d, it is found from Equation 83 that the lateral diffusivity, D_y, would have been estimated at $D_y = 1.1 \times 10^6$ ft^2 Day^{-1} and the vertical diffusivity, D_z, would have been estimated at $D_z = 4700$ ft^2 Day^{-1}. It may be concluded that the estimated lateral diffusivity is smaller than that found from the fitting of the data; and, that the estimated vertical diffusivity is larger than that found from fitting of data. It is possible that this discrepancy is due to buoyancy effects of the heated discharge, if not due to inapplicability of the theoretical relations to the data.

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